B.Sc. Part-I

Paper-I

Theory of Relativity

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Theory of Relativity

Invariance of Momentum under Galilean Transformation:-

Let us Consider the Collision of two bodies. The law of Conservation of Momentum States That in The absence of external forces Sum of momenta before Collision is equal to the Sum of momenta of the bodies after Collision.

If P_1 and P_2 be The momenta Then according to The law of conservation of Momentum.

Momentum before Collision = Momentum after Collision

$$P_1 + P_2 = P_1 + P_2 \quad \text{—— (1)}$$

Also, Momentum P is given by

$$P = mv \quad \text{—— (2)}$$

Derivation:-

Let us assume That The Collision takes place in a region of Space which is far away from external forces. We also assume That The law of Conservation of energy holds.

Let There be two free Particles having velocities u_1 and u_2 in The inertial Position. These two Particles Collide Then after Collision Then velocities becomes v_1 and v_2.

If m_1 and m_2 be The Masses of The Particles. Then before Collision The Kinetic energy of The two Particles

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 \quad \text{—— (3)}$$

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After collision the Kinetic energy

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{—— (4)}$$

Momentum

Then from law of Conservation of energy.

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \Delta W \quad \text{—— (5)}$$

where ΔW = The change in the internal excitation energy of the particles - Consequent to the collision.

This energy may be internal vibrational energy or rotational energy.

In an elastic collision $\Delta W = 0$.

The law of Conservation of momentum holds whether the collision is elastic or inelastic.

Now if we observe the same collision from the system S' moving with respect to S with velocity V then according to Galilean transformation of velocity.

we have,

$$\left.\begin{aligned} u_1' &= u_1 - V \\ u_2' &= u_2 - V \\ v_1' &= v_1 - V \\ v_2' &= v_2 - V \end{aligned}\right\} \quad \text{—— (6)}$$

According to law of Conservation of energy in two systems, we have

energy before collision = energy after collision + ΔW.

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where ΔW = The internal excitation energy which is constant in both the frames

or $\frac{1}{2}m_1 u_1'^2 + \frac{1}{2}m_2 u_2'^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 + \Delta W$ — (7)

where u_1', u_2', v_1' and v_2' represents the corresponding velocities in system S'

Now, on ~~be~~ substituting the values from equ^n (6) we get

$$\frac{1}{2}m_1(u_1 - V)^2 + \frac{1}{2}m_2(u_2 - V)^2 = \frac{1}{2}m_1(v_1 - V)^2 + \frac{1}{2}m_2(v_2 - V)^2 + \Delta W$$

or; $$\frac{1}{2}m_1(u_1^2 + V^2 - 2u_1V) + \frac{1}{2}m_2(u_2^2 + V^2 - 2u_2V) = \frac{1}{2}m_1(v_1^2 + V^2 - 2v_1V) + \frac{1}{2}m_2(v_2^2 + V^2 - 2v_2V) + \Delta W$$

or $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_1V^2 - \frac{\cancel{2}m_1u_1V}{\cancel{2}} + \frac{1}{2}m_2u_2^2 + \frac{1}{2}m_2V^2 - \frac{\cancel{2}m_2u_2V}{\cancel{2}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1V^2 - \frac{\cancel{2}m_1v_1V}{\cancel{2}} + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_2V^2 - \frac{\cancel{2}m_2v_2V}{\cancel{2}} + \Delta W$$

or $$\frac{1}{2}m_1u_1^2 + \cancel{\frac{1}{2}m_1V^2} - m_1u_1V + \frac{1}{2}m_2u_2^2 + \frac{1}{2}m_2V^2 - m_2u_2V = \frac{1}{2}m_1v_1^2 + \cancel{\frac{1}{2}m_1V^2} - m_1v_1V + \frac{1}{2}m_2v_2^2 + \cancel{\frac{1}{2}m_2V^2} - m_2v_2V + \Delta W$$

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$$\text{or} \quad \cancel{\frac{1}{2} m_1 u_1^2 - m_1 u_1 V + \frac{1}{2} m_2 u_2^2 - m_2 u_2 V = \frac{1}{2} m_1 v_1^2}$$

$$- m_1 v_1 V + \frac{1}{2} m_2 v_2^2 - m_2 v_2 V + \Delta W$$

$$\text{or} \quad \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 - (m_1 u_1 + m_2 u_2) V = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \Delta W - (m_1 v_1 + m_2 v_2) V$$

from equ^n (5) $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \Delta W$

$$\therefore \; -(m_1 u_1 + m_2 u_2) V = -(m_1 v_1 + m_2 v_2) V$$

$$\text{or} \quad (m_1 u_1 + m_2 u_2) V = (m_1 v_1 + m_2 v_2) V \quad \text{— (8)}$$

This ~~ep~~ equation helds for all value of V

Therefore, we have

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\text{or} \quad \boxed{(P_1 + P_2)_{before} = (P_1 + P_2)_{after}}$$